Attitude Solutions for a Sun-Pointed, Inertial Spinning Storage Mode Using Unknown Stellar Vectors

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The most significant benefit of on-orbit spacecraft storage is reducing the time in which on-orbit losses to a mission-critical constellation can be mitigated. Acquisition of the normal mission mode attitude is the most limiting factor for the speed with which this can be achieved. This paper describes a technique for recovering the normal, 3-axis stabilized attitude of a spacecraft equipped with fixed star trackers from a 2-axis controlled, Sun-pointing storage mode attitude. Specifically, this technique was developed for a geostationary imaging mission equipped with three Ball CT-602 star trackers. With minimal deviation from the spacecraft's steady-state storage configuration, this technique solves for the unique ECI-to-body quaternion-parameterized attitude using both coarse and fine methods, and is suitable for zero-knowledge acquisition and real-time tracking attitude determination. This technique requires only a minimal change to the current storage mode operations concept, which is to turn on at least two of the three star trackers.

I. Introduction

Spacecraft utilizing an on-orbit storage configuration are typically designed to spend the first two years of their mission life in a 2-axis, spinning, sun-pointed storage mode. This allows a high-degree of flexibility and robustness in the event of mission-profile changes, launch delays for scheduled replacements, on-orbit contingencies[1] and a predetermined, systematic replacement strategy. In the event an operational spacecraft suffers a failure, a stored spacecraft may be returned to normal imaging operations with minimal delay.

Additionally, for spacecraft following strict orbit-control guidelines, an periodic return to the normal mission attitude mode is typically required for stationkeeping operations.

The storage mode exit process can be a complex and time consuming task because of the large number of components powered off. The result is that no on-board attitude knowledge is retained in storage. This fact introduces additional challenge in planning a storage exit, including estimation of solar spin phase and prediction of attitude-dependent telemetry and command coverage and control sensor (star tracker) interference.

Optimization of the storage exit, particularly the determination of the storage mode attitude increases operational efficiency and lowers overall mission risk. Another advantage to this technique is that it can be executed with minimal disturbance to the spacecraft's long-term storage configuration. This permits testing

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and planning activities to begin without committing to a timetable or affecting component life substantially. In this case, the only storage configuration change required is the power-up of two star trackers[a].

This technique consists of two separate estimation methods (coarse and fine), both relying on the traditional q-method. The resultant quantity is a unity-normalized ECI-to-body quaternion. The coarse attitude determination method requires no 3-axis a priori attitude estimate. However, it does require that the spacecraft's controlling sun-sensor axis be sun-pointed, and at least one star vector is provided by the star trackers. Because of the computationally intensive nature of eigenvector problems in general,[2] and the large scale solution-family approach employed, the coarse solution method is suitable only for finding an initial attitude.

Once the initial attitude estimate is obtained, the fine attitude method can be used. The fine method computes a correlative update to the input coarse estimate. Provided that the estimate is within $\sim 2^\circ$ of the correct value, the fine estimator can update attitude estimates quickly enough to track spacecraft attitude in real time. Both the underpinning mathematical theory and a conceptual implementation will be discussed. Most important is that neither relies on differential convergence as do other popular q-method variants.[3]

II. Background

A. Attitude Control Modes

The normal 3-axis attitude control mode for this spacecraft uses a 3-axis rate integrating gyroscope, coupled with a 2-for-3 pair of fixed star trackers. Nominally, a ground-initialized ephemeris provides a True of Epoch vector for the OBC; coupled with an accurate on-board clock, the spacecraft uses the integrated rate from the gyro to predict the position of stars within each tracker's FOV. The OBC then identifies the star based on the on-board catalog and computes a star-tracker frame attitude residual based on the star's predicted position. This residual then serves as the direct input for real-time attitude correction by a 3-axis reaction wheel assembly.

In storage mode, the spacecraft is still under the full control of the reaction wheel assembly, however, a pair of sun sensors serve as the attitude control input. These sensors form a cruciform boresight, with one aligned along a principal axis of the spacecraft. The spacecraft body spins at $0.2\circ$ per second about the sun sensor boresight under tight control from the 2-axis sensor assembly. This attitude provides an environment that is both power positive and thermally safe for an indefinite period of time.

B. Alternate Methods and Motivation

There are several extant attitude determination techniques that lend themselves to the problem above. The TRIAD, QUEST and Optimized TRIAD methods, to name a few, are all acceptable solutions. These were ultimately rejected, in favor of the q-method, for a variety of reasons.

- Variable control over vector weighting
- Non-batch (differential) solution generation
- No dependence on observation covariance (pattern matching)

For this problem, one of the complications is lack of a priori knowledge of the exact source of vector observations. That is, what number of stars will be tracked or available for a given star tracker, or even

[a]The derivations below illustrate all three star trackers in use. Nominally only two of three are in use simultaneously, but all three are polled for data, allowing the algorithm portability between configurations

which star trackers would be used. As is shown in Equation 1, the q-method allows arbitrary weighting of vector observations. For this paper, all weights were taken to be unity, however, this technique allows observations to be arbitrarily biased toward any preferred measurement source. Operationally, this allows the same algorithm to be employed for a fleet of spacecraft, with weighting parameters modified based on past performance for each vehicle.

Secondly, while in storage mode telemetry coverage is not continuous. Additionally, methods that employ differential methods based on batch processing and solution convergence are sensitive to under-sampling. Because of the operational nature of this spacecraft the potential need exists to reliably determine the attitude at any time required.

Finally, many star tracker systems acquire attitude based on some type of pattern-matching algorithm. In fact, this is the default attitude solution behavior for this mission. This technique, though, depends on maximizing points sampled on each tracker (tracking the maximum possible number of stars, preferably bright ones) as well as a well-defined or even fixed inertial position to perform the match. In practical terms, this requires that the spacecraft stop spinning and a perform sequence of directed FOV searches to collect the necessary data. This is a significant departure from the nominal storage configuration and requires a large amount of commanding. For this reason, it is unsuitable for planning activities. Furthermore, this process can be the most time-consuming aspect of a recovery from storage mode to the full 3-axis controlled normal mode, and was thus one of the primary drivers for the development of this technique.

C. Q-Method

The q-method[4] is entirely deterministic and requires simultaneous, multi-frame vector measurements. Specifically, the q-method seeks to minimize the loss function (Eq. 1), given $n \geq 2$ simultaneous vector measurements.[5]

$$J(A) = \sum_{i=1}^{n} w_i \left|\hat{\mathbf{u}}_B^i - A\hat{\mathbf{u}}_R^i\right|^2 \tag{1}$$

Here, A represents the attitude matrix which transforms a vector in the reference frame to the body frame and $\hat{\mathbf{u}}_B^i$ is the unit measurement vector in the body frame. The following summary is repeated from Wertz,[4] though the work was originated by Davenport[5] and Wahba[6] and further extended by Shuster[3] for it's application to MAGSAT. It is possible to differentiate and re-write the loss function in terms of matrices comprised of column vectors:

$$J'(A) = -2\sum_{i=1}^{n} \vec{W}_i A \vec{V}_i + \text{constants} \tag{2}$$

In this case, $\vec{V}_i = \sqrt{w_i}\hat{\mathbf{u}}_B^i$ and $\vec{W}_i = \sqrt{w_i}\hat{\mathbf{u}}_R^i$ such that:

$$W = \left[W_1 \vdots \cdots \vdots W_n\right] \tag{3}$$
$$V = \left[V_1 \vdots \cdots \vdots V_n\right]$$

The derivative of the loss function then becomes

$$J'(A) \equiv tr(W^T A V) \tag{4}$$

By re-parameterizing the attitude matrix in terms of the unit quaternion, such that $A = A(\vec{q})$, yields the far more convenient form of the loss function derivative (Eq. 5).

$$J'(\vec{q}) = \vec{q}^T K \vec{q} \tag{5}$$

The rather tedious definitions for the 4×4 K-matrix and its constituents follow.

$$K = \begin{pmatrix} S - \mathbf{1}\sigma & \mathbf{Z} \\ \mathbf{Z}^T & \sigma \end{pmatrix} \tag{6}$$

$$B \equiv WV^T \tag{7}$$

$$S \equiv B^T + B \tag{8}$$

$$Z \equiv (B_{2,3} - B_{3,2}, B_{3,1} - B_{1,3}, B_{1,2} - B_{2,1})^T \tag{9}$$

$$\sigma \equiv tr(B) \tag{10}$$

After considerable algebraic manipulation (See,[5] Appendix C.), which will not be shown here, the eigenvector equation (Eq. 11) for K can be extracted.

$$K\vec{q} = \lambda\vec{q} \tag{11}$$

Substituting this back into Eq. 5, shows

$$J'(\vec{q}) = \vec{q}^T K\vec{q} = \vec{q}^T \lambda\vec{q} = \lambda \tag{12}$$

Therefore, the eigenvector corresponding to the minimum of the loss function, maps directly to the maximum eigenvalue. These eigenvectors are attitude quaternions, the quaternion with the highest corresponding eigenvalue is the attitude quaternion which represents the best fit attitude parameterizing the Reference-to-body rotation.

For the purpose of this paper, I choose the Reference frame, to be the Earth-Centered Inertial, True of Date frame[b]. The sun vector (both in the body and ECI frames) provides the first reference point for this technique. In both the coarse and fine methods, arbitrary stellar vectors complete the family of observations.

As G.M. Lerner[4] noted, in his description of this technique, the complication arises not from body-frame observations, but in adequately structuring vector measurements in the corresponding reference (ECI) frame[c]. The construction and determination of ECI vector observations is the primary occupation of this paper.

D. Characteristics of the Ball CT-602 Star Trackers

Ball star trackers return a horizontal and vertical coordinate (”H” and ”V”) for the position of a star on the CCD focal plane, along with an apparent magnitude for up to five stars per tracker. As part of this system, there is a large star catalog listing apparent magnitude, ECI vector and ID number.

Due to the inherent difficulty in measuring apparent magnitude with this type of instrument, the measured

[b]Hereafter referred to simply as ”ECI.”

[c]Lerner also discusses the numerical rigor involved in the computation of eigenvector/eigenvalue pairs. While this is not to be underestimated, even in the modern computational age, freely available linear algebra packages such as ATLAS are able place techniques such as this within easy effective reach.

magnitude serves largely as a discriminator for the identification of stars, and is therefore very loosely constrained. Instead, the precise location on the CCD is tightly constrained in this operation.

Because the CCD coordinate returned is two dimensional, we must first scale and transform it back to a three-dimensional unit vector, before it can be used in any 3-D rotational computations.

$$\epsilon_i = \left(h_i^2 + v_i^2 + 1\right)^{-\frac{1}{2}} \tag{13}$$

$$\rho_i = \begin{pmatrix} \epsilon_i h_i \\ \epsilon_i v_i \\ \epsilon_i \end{pmatrix} \tag{14}$$

This parameterization, Eq. 14 yields a unit vector ρ_i representing a given star's measured position in the star tracker reference frame.

III. Coarse Attitude Determination

This method relies on the computation of large families of eigenvectors comprising a complete eigenspace, on which lies a theoretical single-valued solution. In practical terms, this amounts to the computation of a large family of possible attitude solutions derived from all star measurements, eliminating those that do not conform to all vector observations, and then finding a suitable mean (or filtered value) for the remaining observations.

A. Star Catalog Subsetting

The first task, is to parse the full star catalog for only the stars in each star tracker's possible field of view. As a spin-axis aligned spacecraft, for the current ECI sun vector, $\hat{s}_{ECI}$, each star tracker's possible star sub-catalog is constrained to a sector described by their effective FOV (δ), and the cone of rotation defining the star tracker's orientation relative to the spin axis in the body frame.

First, we find the angles α_i between the each star tracker boresight and the sun vector coincident spin axis in the body frame.

$$\alpha^n = \arccos(\hat{b}^n \cdot \hat{s}_b) \tag{15}$$

We then isolate all the stars' (φ) catalog ECI vectors, $\Sigma(\varphi_i)$, in the star catalog for each star tracker that meet this angular constraint.

$$\varphi^n \in [\alpha^n + \delta \geq |\Sigma(\varphi_i) \cdot \hat{s}_b|] \tag{16}$$

Therefore, we find a set of stars $\varphi^{1,2,3}$ available to each star tracker for a given sun vector.

B. Building Up Solution Families

With the stars in each annular band selected, observations can now be processed. I will denote the set of measured magnitudes as $\gamma(\varphi_i)$, and catalog magnitudes as $\Gamma(\varphi_i)$. In a given star tracker, n, the band sub-catalog is searched for a family of stars cataloged near the magnitude of each measured star (m) magnitude. For this implementation, I selected a tolerance of ± 0.5 magnitude.

$$\gamma^n(\varphi_i) \pm 0.5 \in \Gamma^n(\varphi) \tag{17}$$

This relation gives a set of stellar candidates for each observation, based on magnitude: $\gamma^n(\varphi_i^m)$. Because each of these stellar candidates represents a known, cataloged star, the task is to now construct observation and reference matrices (Eq. 3), using the measured star tracker coordinate (Eq. 14) for the observation matrix (W)

$$W = \omega_W \left(\hat{s}_b \;\vdots\; M_{b-STn} \cdot \rho_i \right) \tag{18}$$

and each stellar candidate from the catalog to build a unique reference matrix

$$V = \omega_V \left(\hat{s}_{ECI} \;\vdots\; \Gamma\left(\varphi_i^m\right) \right) \tag{19}$$

where the variables ω_W and ω_V are the square roots of the normalization coefficients[d] referenced in Eq. 1. Let M_{b-STn} be the matrix defining the rotation of a vector between the spacecraft body and star tracker n frames.

With a fixed observation matrix for each particular star measurement (W_i), eigenvector/eigenvalue sets are then produced for it and each candidate reference matrix (V_i^m). These combinations produce a family of matrices, $K^{i,m}$ as outlined in Eqs. 7 through 11. In each case, the eigenvector corresponding to the largest nonzero eigenvalue is retained in a large data set.

$$\mathbf{Q}^{i,m} = \text{eigenvectors}\left(K^{i,m}\right)|_{\max\,\lambda} \tag{20}$$

The process is then repeated for all measured stars in all star trackers, adding to $\mathbf{Q}^{i,m}$.

C. Solution Isolation and Condensation

With the large eigenspace $\mathbf{Q}^{i,m}$ fully mapped, what remains is to validate each quaternion against observations. The crux of creating the eigenspace is to focus on each observed star in isolation, creating a probable solution vector for each possible catalog star representing that measurement. In theory, this process should produce at least one accurate solution for each observation, and that the mutual solutions for each observation should be numerically similar.

The criteria for validating those solutions, is that they accurately reproduce *all* observables (stellar position and magnitude) to within accurate tolerances. These constraints depend strongly on star position ($\leq 0.2°$), and weakly on magnitude (± 0.5).

First, the star tracker band subcatalog (ECI vectors, $\Sigma^n(\varphi)$), are further reduced to a list of stars only in the probable field of view, $\Sigma^n(\varphi^{i,m})$[e]. The simplest way, is to transform all possible star vectors in this subsample into the star tracker frame (Eq. 21) and then dot the k observation vectors (Eq. 23) into the result.

$$\Sigma_b^n(\varphi^{i,m}) = A\left(\mathbf{Q}^{i,m}\right)\Sigma^n(\varphi^{i,m}) \tag{21}$$

$$\Sigma_{ST}^n(\varphi^{i,m}) = M_{STn-b}\Sigma_b^n(\varphi^{i,m}) \tag{22}$$

$$\varrho^n = \Sigma_{ST}^n(\varphi^{i,m})\left(\rho_1 \;\vdots\; \dots \;\vdots\; \rho_k\right) \tag{23}$$

The matrix ϱ^n gives the cosines of the residuals of each possible star (rows) to each observation (columns). Checking the spatial constraint involves finding the maximum value of each column and checking to ensure that the values are $\geq \cos(0.2°)$. The magnitude check is then applied for validation. Quaternion solutions that pass this check are binned and then a mean is taken when all evaluations are complete.

[d] Depending on the technique used to extract eigenvectors, these coefficients may be irrelevant aside from scaling the eigenvalues.

[e] That is, the star tracker fields of view corresponding to a possible attitude quaternion

It is worth noting that the act of finding the quaternion mean can take on numerous dimensions of complexity, depending on the physical significance the user wishes to enforce. In the case of the exemplar spacecraft, only a direct, normalized numerical mean (by quaternion element) was used. Other options, however, exist. In the case of the exemplar these could include: mean spin phase for a fixed solar vector or tilted plane averaging (to account for misalignments between trackers), for example.

IV. Fine Attitude Tracking

IN the fine solution, the initial attitude estimate q_{est}[f] is used to find stars in the corresponding fields of view, with an additional margin factor. A residual matrix (Eq. 23) is then computed. The maximum values for each column (the observations) are then correlated statistically. This ensures that, while their values may differ, the deviation is mutual. Physically, this should prove smooth secular motion from the initial estimate to the updated attitude. If the observed star vectors correlate[g], the stars are identified and concatenated into W and V matrices (Eq. 3). This process is repeated for all observations on all star trackers. The q-method is then applied simultaneously to all observations, using the unified W and V matrices, and producing a single valued result.

A. Estimated Catalog Field of View

To subset the full star catalog to an estimated field of view, we first downselect the band catalogs to include the stars within some margin of the nominal field of view. Based on the nominal secular motion in storage, a factor of 1.2 was selected.

$$\varphi_i^n \in [\alpha^n + 1.2\delta \geq |\Sigma(\varphi_i) \cdot \hat{s}_b|] \tag{24}$$

We then compute the star tracker frame vectors for these stars, along with the estimated residual matrix.

$$\Sigma_b^n(\varphi_i^n) = A\left(\mathbf{q}_{est}\right)\Sigma^n(\varphi_i^n) \tag{25}$$

$$\Sigma_{ST}^n(\varphi^{i,m}) = M_{STn-b}\Sigma_b^n(\varphi^{i,m}) \tag{26}$$

$$\varrho_{est}^n = \Sigma_{ST}^n(\varphi_i^n)\begin{pmatrix}\rho_1 & \vdots & \dots & \vdots & \rho_k\end{pmatrix} \tag{27}$$

B. Correlation and Concatenation

The maximum values in each column of ϱ_{est} are then correlated as a one dimensional sample, such as in[7] (p. 719). Because of the precision required, we demand a correlation coefficient in excess of 0.99999.

If this criterion is satisfied, the appropriate stars are identified. Their observation vectors are transformed into the body frame, and the corresponding body and ECI vectors are concatenated into the aggregate W and V matrices

$$W = \begin{pmatrix}\mathbf{s}_b & \left(M_{b-ST1}\rho_1^1\right)^T & \vdots & \dots & \vdots & \left(M_{b-ST3}\rho_k^3\right)^T\end{pmatrix} \tag{28}$$

$$V = \begin{pmatrix}\mathbf{s}_{ECI} & \Sigma^3(\varphi_1^1) & \vdots & \dots & \vdots & \Sigma^3(\varphi_k^3)\end{pmatrix} \tag{29}$$

for all observables on all three star trackers.

[f] If it is possible to assume low-rate secular motion following the coarse attitude acquisition, it is suitable to use that output as the first estimate to the fine attitude tracker

[g] Programmatically, if these vectors do not correlate, an error flag should be returned. In this instance, the coarse attitude acquisition must be re-run as if this were an initial confused-in-space scenario.

C. Tracked Solution

The aggregate observation W-matrix (Eq. 28) and reference V-matrix (Eq. 29) are then transformed to an aggregate K-matrix (Eq. 6) via Eqs. 7-10. Invoking Eq. 20 yields a fine, single-valued attitude solution.

Conclusion and Notes for Further Development

On most modern computers, the fine solution technique can track faster than the update interval for most telemetry. It is suitable for real-time, or near real-time monitoring of evolving spacecraft attitude. For this reason, it makes possible not only attitude determination for storage evaluation and exit planning (Fig. 2), but also opens possibilities for routine monitoring. Capturing an accurate time-evolving attitude estimate while on-station may be of interest for both bus characterization and anomaly investigation.

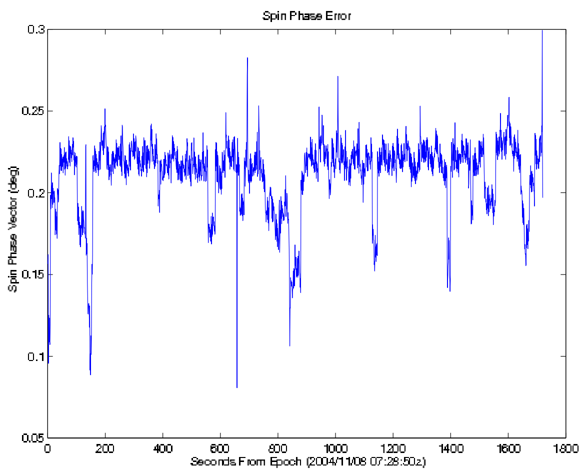

Figure 1. Error in computing the spin phase vector about the ECI sun vector.

The demand of multiple, simultaneous (deterministic) vector measurements restricts potential applications of the above technique to a small subset of stored spacecraft.

The results of this technique are unfiltered (Fig. 1). Useful extensions of the above technique would be in the application of relevant averaging for the coarse method and state-estimation filtering for the fine method. Of specific note is the problem of characterizing and correcting tilted-plane CCD and boresight misalignments between star trackers in coarse attitude estimation. Additionally, the application of state estimation methods (such as a Kalman filter) could dramatically improve the solution stability and convergence of the fine estimation method.

In conclusion,, the application of a direct technique that is both efficient and non-differential in time provides a potentially significant capability for the operators of stored spacecraft in general, and NOAA in particular. While this technique is not yet proven using flight data, it has been successfully validated in simulation and is ready for operational testing.

Appendix I

Below is a plot illustrating the performance of the method under simulated conditions. Figure 2, below, shows the error in faster than real-time modeling the sun vector's position with the simple SMPOS algorithm. This accounts for a significant portion of the measured error in spin phase. (Fig. 1). In this example case, the star trackers are mounted with axes perpendicular to this spin axis–therefore, the estimated error in the spin axis phase represents a sensitive characterization of the overall estimated attitude error. In any event, the solutions are fine enough for both planning and spacecraft attitude acquisition purposes.

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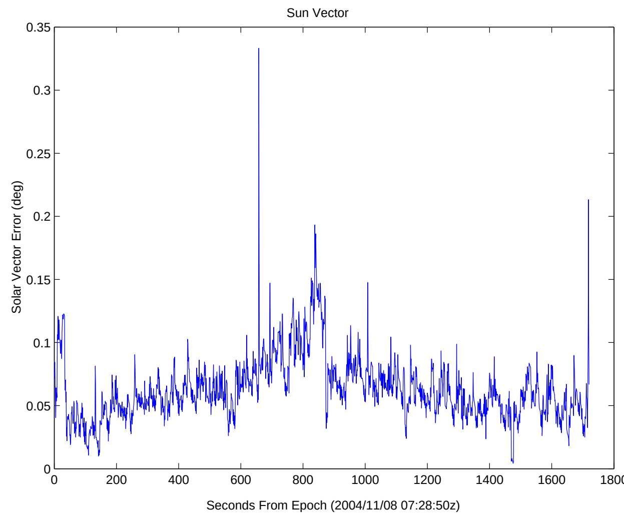

Figure 2. ECI sun vector pointing error. The solar vector was simulated SMPOS and compared to true data